7-2 Maxwell's Equations for Magnetostatics

Reading Assignment: *pp. 205-207*

Recall that the static form of Maxwell's Equations decoupled into Electrostatic and Magnetostatic equations.

It's now time to consider the **Magnetostatic Equations!**

HO: Maxwell's Equations for Magnetostatics

Q:

A: HO: The Integral Form of Magnetostatics

Maxwell's Equations for Magnetostatics

From the **point form** of Maxwell's equations, we find that the **static** case reduces to another (in addition to electrostatics) pair of **coupled differential equations** involving magnetic flux density $\mathbf{B}(\bar{r})$ and current density $\mathbf{J}(\bar{r})$:

$$\nabla \cdot \mathbf{B}(\bar{r}) = 0 \qquad \nabla x \mathbf{B}(\bar{r}) = \mu_0 \mathbf{J}(\bar{r})$$

Recall from the **Lorentz force equation** that the magnetic flux density $\mathbf{B}(\bar{r})$ will apply a **force** on current density $\mathbf{J}(\bar{r})$ flowing in volume dv equal to:

$$\mathbf{dF} = \left(\mathbf{J}(\bar{r}) x \mathbf{B}(\bar{r})\right) dv$$

Current density $\mathbf{J}(\bar{r})$ is of course expressed in units of **Amps/meter^2**. The units of magnetic flux density $\mathbf{B}(\bar{r})$ are:

$$\frac{\text{Newton} \cdot \text{seconds}}{\text{Coulomb} \cdot \text{meter}} \doteq \frac{\text{Weber}}{\text{meter}^2} \doteq \text{Tesla}$$

* Recall the units for **electric** flux density $\mathbf{D}(\bar{r})$ are **Colombs/m^2**. **Compare** this to the units for **magnetic** flux density—Webers/m^2.

* We can say therefore that the units of **electric** flux are **Coulombs**, whereas the units of **magnetic** flux are **Webers**.

* The concept of **magnetic flux** is much more important and useful than the concept of electric flux, as there is **no** such thing as **magnetic charge**.

We will talk much more later about the concept of **magnetic flux**!

Now, let us consider specifically the **two** magnetostatic equations.

* First, we note that they specify both the **divergence** and **curl** of magnetic flux density $\mathbf{B}(\bar{r})$, thus **completely** specifying this vector field.

* Second, it is apparent that the magnetic flux density $\mathbf{B}(\bar{r})$ is **not conservative** (i.e, $\nabla x \mathbf{B}(\bar{r}) = \mu_0 \mathbf{J}(\bar{r}) \neq 0$).

* Finally, we note that the magnetic flux density **is a solenoidal** vector field (i.e., $\nabla \cdot \mathbf{B}(\bar{r}) = 0$).

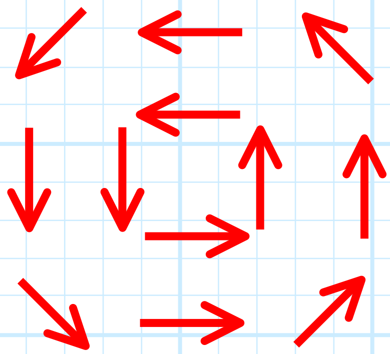
Consider the **first** of the magnetostatic equations:

$$\nabla \cdot \mathbf{B}(\bar{r}) = 0$$

This equation is sometimes referred to as **Gauss's Law for magnetics**, for its obvious **similarity** to Gauss's Law of electrostatics.

This equation essentially states that the magnetic flux density does **not diverge** nor converge from any point. In other words, it states that there is no such thing as **magnetic charge** !

This of course is **consistent** with our understanding of **solenoidal** vector fields. The vector field will **rotate** about a point, but not diverge from it.



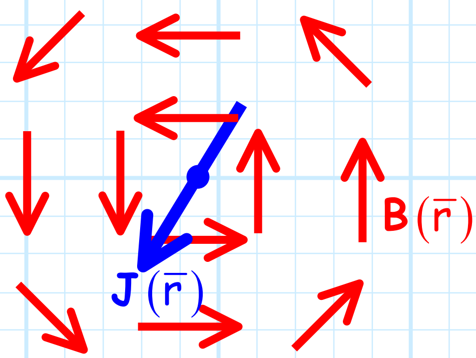
Q: *Just what* ***does*** *the magnetic flux density* $\mathbf{B}(\bar{r})$ *rotate around ?*

A: Look at the **second** magnetostatic equation!

The **second** magnetostatic equation is referred to as **Ampere's Circuital Law**:

$$\nabla x \mathbf{B}(\bar{r}) = \mu_0 \mathbf{J}(\bar{r}) \quad \text{Ampere's Law}$$

This equation indicates that the magnetic flux density $\mathbf{B}(\bar{r})$ **rotates around** current density $\mathbf{J}(\bar{r})$ --the **source** of magnetic flux density is current!.



The Integral Form of Magnetostatics

Say we evaluate the **surface integral** of the point form of **Ampere's Law** over some arbitrary surface S.

$$\iint_S \nabla \times \mathbf{B}(\bar{\mathbf{r}}) \cdot \overline{ds} = \mu_0 \iint_S \mathbf{J}(\bar{\mathbf{r}}) \cdot \overline{ds}$$

Using **Stoke's Theorem**, we can write the **left** side of this equation as:

$$\iint_S \nabla \times \mathbf{B}(\bar{\mathbf{r}}) \cdot \overline{ds} = \oint_C \mathbf{B}(\bar{\mathbf{r}}) \cdot \overline{d\ell}$$

We also recognize that the **right** side of the equation is:

$$\mu_0 \iint_S \mathbf{J}(\bar{\mathbf{r}}) \cdot \overline{ds} = \mu_0 \mathrm{I}$$

where I is the **current** flowing through surface S.

Therefore, combing these two results, we find the integral form of **Ampere's Law** (Note the **direction** of I is defined by the **right-hand rule**):

$$\oint_C \mathbf{B}(\bar{\mathbf{r}}) \cdot \overline{d\ell} = \mu_0 I$$

Amperes law states that the **line integral** of $\mathbf{B}(\bar{r})$ around a **closed contour** C is proportional to the **total current** I flowing **through** this closed contour ($\mathbf{B}(\bar{r})$ is **not** conservative!).

Likewise, we can take a **volume integral** over both sides of the magnetostatic equation $\nabla \cdot \mathbf{B}(\bar{r}) = 0$:

$$\iiint_V \nabla \cdot \mathbf{B}(\bar{r})\, dv = 0$$

But wait! The left side can be rewritten using the **Divergence Theorem**:

$$\iiint_V \nabla \cdot \mathbf{B}(\bar{r})\, dv = \oiint_S \mathbf{B}(\bar{r}) \cdot \overline{ds}$$

where S is the **closed surface** that **surrounds** volume V. Therefore, we can write the integral form of $\nabla \cdot \mathbf{B}(\bar{r}) = 0$ as:

$$\oiint_S \mathbf{B}(\bar{r}) \cdot \overline{ds} = 0$$

Summarizing, the **integral form** of the magnetostatic equations are:

$$\oiint_S \mathbf{B}(\bar{r}) \cdot \overline{ds} = 0 \qquad \oint_C \mathbf{B}(\bar{r}) \cdot \overline{d\ell} = \mu_0 I$$